The Parallelism in Goedel’ s Philosophy of Mathematics

Gao Kun

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Introduction

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- However, some modification and development has been made because of my further reading and thinking on the topic after writing that paper.
- The terminology parallelism here has nothing to do with the standard use as referring to the metaphysical doctrine about mind and body.I employ it to name Goedel's thought contained in his analogy between mathematics and natural science, which is a salient feature of Goedel's philosophy and so rich that it deserves a special term.

Introduction

- To appreciate the richness of this analogy, we only need to have a look at a passage from Goedel, 1944, where he traces the analogy to Russell:
  He compares the axioms of logic and mathematics with the laws of nature and logical evidence with sense perception, so that the axioms need not necessarily be evident in themselves, but rather their justification lies(exactly as in physics)in the fact that they make it possible for these "sense perceptions" to be dedueced; which of course would not exclude that they also have a kind of intrinsic plausibility similar to that in physics.

Introduction

- From this short passage, we already can see Goedel’s analagy between mathematics and science is not just a trivial metaphor, which expresses his Platonistic stance in mathematics. On the contrary, it includes an intriguing epistemological espect, involving the justification of axioms. And even for the ontological aspect, it is beyond the mere affirmation of realism, but rather constitutes an argument for realism, which is of the so called Fregean form. To explore all these aspects and examine their consequences and problems is just what we are going to do in the following.

Introduction

Some works among other references we will heavily depend on are:

- Kurt Goedel: *Russel's mathematical logic*, 1944; *What is Cantor' s continuum problem?*, 1947, 1964; *Some basic theorems on the foundations of mathematics and their implications*, *1951; *Is mathematics syntax of language?*, *1953/9.
- Charles Parsons: *Platonism and mathematical intuition in Goedel's thought*,1995.
- Donald A. Martin: *Goedel's conceptual realism*, 2005.

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- Penelope Maddy: *Realism in Mathematics*, 1990; *Naturalism in mathematics*,1997; *Defending the axioms*, 2011.
- W.V.O Quine: *Word and Object*, 1960.
- 郝兆宽：《哥德尔的数学哲学》，2008.

Parallelism as an argument for Platonism

In his 1944, Goedel writes:

It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory system of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions and in both cases it is impossible to interpret the propositions one wants to assert about these entities as propositions about the "data", i.e., in the latter case the actually occurring sense perceptions.

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Recall that Quine had a similar view about mathematical objects, i.e. taking them existing objectively for the same reason as taking physical objects,such as stones and electrons, existing objectively. So, what is the difference?

Parallelism as an argument for Platonism

There is a general form of argument for the existence of mathematical objects, which is traced back to Frege and referred to as the Fregean argument(cf. Linnebo, ?ystein, "Platonism in the Philosophy of Mathematics", The Stanford Encyclopedia of Philosophy <http://plato.stanford.edu/archives/fall2011/entries/platonism-mathematics/>):

1. Semantic premise: The singular terms of the language of mathematics purport to refer to mathematical objects, and its first-order quantifiers purport to range over such objects.

2. Truth premise: Most sentences accepted as mathematical theorems are true.

3. Conclusion: Mathematical objects exist.

Parallelism as an argument for Platonism

Clearly, Quine’s indispensibility argument for realism is along this line: by refuting the possibility of constructing a nominalistic language for science, he alleged we are committed to the existence of abstract objects.

And so is Goedel’s paralllelism argument: ”it is impossible to interpret the propositions one wants to assert about these entities as propositions about the ”data”, i.e., in the latter case the actually occurring sense perceptions”.

Parallelism as an argument for Platonism

But the difference is also not hard to see:

Goedel argues for realism by claiming its necessity for a "satisfactory" system of mathematics, which can be temporally identified with classical mathematics, since to obtain the latter we need to refer to classes(Russell's "no class theory" is unsuccessful), use impredicative definitions and etc.

In contrast, Quine's argument is based on the utility of abstract objects for natural science: to assume the existence of numbers is to obtain a theory of sense perceptions, just like to assume electrons.

Parallelism as an argument for Platonism

Two consequences:

- Quine can reduce the Truth premise to the truth of natural science, while Goedel cannot.
- Goedel's respect for the autonomy of mathematics leads him to a quite different attitude to the axiom of constructibility: $V = L$, which is highly more consistent with mathematical practice than that of Quine.

Problems of the argument

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I would like to raise two questions about Goedel's parallelism argument for Platonism.Each of them involves a central concept of Goedel's philosophy, i.e. "intuition" and "concept" respectively:

1. What is the mathematical data parallel with the sense data?

2. Does the argument supports the existence of concepts as intensional objects?

Problems of the argument

For the first question, a famous passage from 1964 may be quoted:

It should be noted that mathematical intuition need not be conceived of as a faculty giving an immediate knowledge of the objects concerned. Rather it seems that, as in the case of physical experience, we form our ideas also of those objects on the basis of something else which is immediately given. Only this something else here is not, or not primarily, the sensations. That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively diffrent from sensations . . .

Problems of the argument

It can be seen that Goedel did think there is parallel data: a kind of intuition of mathematical objects. However, he did not provide a sufficient account for it.

Problems of the argument

For the second question, it should be noted that to express classical mathematics we only need to refer to extensional objects like numbers, sets and etc, rather than intensional objects like concepts, so the parallelism argument is not going to work for concepts. But on the other hand, Goedel attaches much importance to the objective existence of concepts, and often talks of our intuition of , e.g., the concept of sets, rather than sets themselves.

Problems of the argument

One possible way out is to dig the nature of Goedelian concepts, and it is even more attractive when we notice the unusual use of the term by Goedel: he often talks about set theoretic axioms implied by the concept of sets, since this does not seem to be possible considering that many axioms are existential.

Problems of the argument

Two interpretations:

- Martin: sometimes, the instances of Goedelian concepts are structures rather than individual objects.
- 郝兆宽：“所谓集合的概念就是集合的论域，是集合构成的一个世界，就像原子构成了一个论域一样。”

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If we follow Hao, the difficulty can be easily overcome, but:

1. In 1944, Goedel explicitly distinguish classes and concepts, interpreting the former as "pluralities of things or structures consisting of a plurality of things",and the latter as "the properties and relations of things existing independently of our definitions and constructions".

2. The Goedelian analytic-sythetic distinction would be fraudulent.

3. More considerations.

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Goedel's epistemology consists of two parts: mathematical intuition and the extrinsic method of justification for axioms.

Mathematical intuition is compared with sense perception, since for Goedel it has the following characteristics which are also usually attributed to sense perception:

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Question: particular belief vs general belief Answer: The object of intuition is concept rather than particular object.

A methodology parallel to the hypothesis-deduction model in natural science

Goedel introduces the inductive method into the defence of axioms in his 1947:

. . . even disregarding the intrinsic necessity of some new axiom, and even in case it has no intrinsic necessity at all, a probable decision about its truth is possible also in another way, namely inductively by studying its "success". Success here means fruitfulness in consequences, in particular in verifiable consequences, i.e., consequences demonstrable without the new axiom, whose proofs with the help of the new axiom, however, are considerably simpler and easier to discover, and make it possible to contract into one proof many different proofs . . .

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. . . there might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, and yielding such powerful methods for solving problems (and even solve them constructively, as far as that is possible) that, no matter whether or not they are intrinsically necessary, they would have to be accepted at least in the same sense as any well-established physical theory.

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In the above passage, we see that Goedel compares extrinsic justifications for axioms to theoretical justifications in physical theory. The logic here is : if when we study the physical world induction can be used, why it cannot when we study the conceptual world, which is also an objective realm as the physical one? We postpone the examination of the legitimacy of this methodology, and consider another problem firstly:

Under Goedel's epistemology, is mathematical knowledge a priori or a posteriori?

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Assuming the traditional definition of apriority, which takes a priori knowledge as that independent of experience, we claim that the introduction of inductive method does not undermine apriority of mathematics.

Demonstration:

e.g. All ravens are black.

The observational part and inductive step.

A posteriority and fallibility.

The legitimacy of the extrinsic justification: challenge from Maddy

Goedel proposed the extrinsic method of justification primarily as a way to treat independent questions, such as the Continuum Problem.

It has been practiced by contemporary set theorists to a high degree. Much work at two levels has been done:

- examining consequences of both CH and $\neg CH$;
- examining consequences of various axiom candidates, such as PD, which may be able to decide CH.

The legitimacy of the extrinsic justification: challenge from Maddy

The general form of set theorists' argument for an Axiom candidate:

It results a theory very nice. e.g. PD is preferable to V=L, because the former provides a nice decision for many independent questions in descriptive set theory, and the latter is so restrictive.

But why the real world of sets should be nice? We do not ask the physical world to satisfy our wishes, and what we like is not always what is true.

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Reconsidering the validity of the Goedelian extrinsic method of justification under Platonistic philosophy.

Further remarks

- Maddy’s turn from set theoretic realism to naturalism

Further remarks

- Maddy’s turn from set theoretic realism to naturalism
- My attitude change

Thanks for your attention!